量化 Quiz 1、2015.3.9

1、$\rho(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1}d\nu$ (40%)

$\Rightarrow \rho(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{e^{hc/\lambda kT}-1}\right)d\lambda$

2、(a) $\lambda_{max}T = \frac{hc}{4.965k}$ (40%) $k = 1.381\times10^{-23}\ JK^{-1}$

(b) $T = 10000K$, $\lambda_{max} = ?$ (20%) $c = 2.998\times10^{8}\ ms^{-1}$

ANS:

1、$c = \nu\cdot\lambda \Rightarrow \nu = \frac{c}{\lambda} \Rightarrow \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2} \Rightarrow d\nu = -\frac{c}{\lambda^2}d\lambda$

在此物通常取其大小 $|d\nu| = \frac{c}{\lambda^2}d\lambda$

$\rho(\nu)d\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1}d\nu = \frac{8\pi h}{c^3}\frac{(\frac{c}{\lambda})^3}{e^{hc/\lambda kT}-1}\frac{c}{\lambda^2}d\lambda$

$\rho(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{e^{hc/\lambda kT}-1}\right)d\lambda$ #

2、(a) $\lambda_{max} = \rho'(\lambda) = \frac{\rho(\lambda)}{d\lambda} = 0$ 求出 λ ($k = 1.38\times10^{-23}\ JK^{-1}$, $c = 2.998\times10^{8}\ ms^{-1}$)

$\rho(\lambda) = \frac{8\pi hc}{\lambda^5}\left(\frac{1}{e^{hc/\lambda kT}-1}\right)$, 令 $\frac{hc}{\lambda\cdot k\cdot T} = x$, $\lambda = \frac{hc}{x\cdot k\cdot T}$

$dx = \frac{-hc}{kT}\cdot\frac{1}{\lambda^2}d\lambda = \frac{-hc}{kT}\left(\frac{x\cdot k\cdot T}{h\cdot c}\right)^2 d\lambda = \left(\frac{-kTx^2}{hc}\right)d\lambda$

$\frac{d\rho(\lambda)}{d\lambda} = \frac{d\rho(\lambda)}{dx}\cdot\frac{dx}{d\lambda}$ (by chain rule)

$= \left(\frac{-kT}{hc}\right)x^2\cdot\frac{d\rho(\lambda)}{dx}$

$$= \left(\frac{-kT}{hc}\right) x^2 \cdot \frac{d}{dx}\left[(8\pi hc)\left(\frac{xkT}{hc}\right)^5 (e^x-1)^{-1}\right] = 0$$

$$= \underbrace{-\left(\frac{kT}{hc}\right)^6 \cdot 8\pi hc}_{\text{常数}} \cdot x^2 \cdot \frac{d}{dx}\left[x^5 (e^x-1)^{-1}\right] = 0$$

$$\Rightarrow \frac{d}{dx}\left[x^5 (e^x-1)^{-1}\right] \Rightarrow \left[5x^4 (e^x-1)^{-1} - x^5 \cdot e^x (e^x-1)^{-2}\right] = 0$$

$$\frac{5x^4}{(e^x-1)} - \frac{x^5 e^x}{(e^x-1)^2} = 0$$

$$\frac{5x^4(e^x-1) - x^5 e^x}{(e^x-1)^2} = 0 \Rightarrow 5x^4(e^x-1) - x^5 e^x = 0 \quad (\text{同除} x^4)$$

$$\Rightarrow 5e^x - 5 - xe^x = 0$$

$$\Rightarrow xe^x = 5e^x - 5$$

利用数值法，求 x，$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_n = 7 \Rightarrow x_{n+1} = 6.33$

$x_n = 6 \Rightarrow x_{n+1} = 5.49$

$x_n = 5 \Rightarrow x_{n+1} = 4.96$

$\vdots$

$x \doteq 4.965$

代回 $x = \frac{hc}{\lambda \cdot k \cdot T} \Rightarrow \lambda_{max} T = \underline{\frac{hc}{4.965k}}$

(b) $T = 10000\,K$

$$\lambda_{max} = \frac{6.626 \times 10^{-34} \times 2.998 \times 10^8}{4.965 \times 1.381 \times 10^{-23} \times 10000}$$

$$= 289.7\,(nm)$$