1. damped oscillation (50%)

m=10 kg , c=20 kg/sec, k=90 nt/m

ANS:

$$my''+cy'+ky=0$$

$$\Rightarrow 10y''+20y'+90y=0$$

$$\Rightarrow y''+2y'+9y=0$$

令$y=e^{\lambda x}$, $y'=\lambda e^{\lambda x}$, $y''=\lambda^2 e^{\lambda x}$

$$\lambda^2+2\lambda+9=0$$

$$\lambda=\frac{-2\pm\sqrt{4-36}}{2}=-1\pm 2\sqrt{2}i$$

$$y(t)=e^{-t}(A\cos(2\sqrt{2})t+B\sin(2\sqrt{2})t)$$

$$y(0)\Rightarrow A=0.16$$

$$y'(0)\Rightarrow$$

$$y(t)=e^{-t}(0.16\cos(2\sqrt{2})t+B\sin(2\sqrt{2})t)$$

$$y'(t)=-e^{-t}\cdot 0.16\cos(2\sqrt{2})t+-e^{-t}\cdot B\sin(2\sqrt{2})t$$
$$+0.16\cdot 2\sqrt{2}\cdot e^{-t}\cdot -\sin(2\sqrt{2})t$$
$$+e^{-t}\cdot 2\sqrt{2}\cdot B\cos(2\sqrt{2})t$$

$$y'(0)=0.16+2\sqrt{2}B=0$$

$$B=\frac{0.16}{2\sqrt{2}}=\frac{\sqrt{2}}{25}$$

$$y(t)=e^{-t}(0.16\cos(2\sqrt{2})t+\frac{\sqrt{2}}{25}\sin(2\sqrt{2})t)$$

2. particle in a box (50%)

(i)寫出 Schrodinger Equation

(ii)求解 E? ψ ?

$$-\frac{\hbar^2}{2m}\psi''(x)+V(x)\psi(x)=E\psi(x)$$

$$-\frac{\hbar^2}{2m}\psi''(x)=E\psi(x)$$

$$\psi''=-(\frac{2mE}{\hbar^2})\psi=-k^2\psi$$

$$\psi=A\cos kx+B\sin kx$$

$$\psi(0)=0,\psi(L)=0$$

$$A=0,B\sin kL=0,kL=n\pi,k=\frac{n\pi}{L}$$

$$\psi=B\sin\frac{n\pi x}{L},\frac{2mE}{\hbar^2}=\frac{n^2\pi^2}{L^2}$$

$$E=\frac{n^2h^2}{8mL^2}$$