

Quiz 5

1. $(1+x)y' = 2y$

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots$$

$$y' = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \dots$$

$$2y = 2(a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots)$$

$$(1+x)y' = (a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \dots)$$

$$+ (a_1x + 2a_2x^2 + 3a_3x^3 + 4a_4x^4 + 5a_5x^5 + \dots)$$

$$(1+x)y' = 2y$$

$$x^0 \Rightarrow a_1 = 2a_0$$

$$x^1 \Rightarrow 2a_2 + a_1 = 2a_1, \quad 2a_2 = a_1, \quad a_2 = a_0$$

$$x^2 \Rightarrow 3a_3 + 2a_2 = 2a_2, \quad a_3 = 0$$

$$x^3 \Rightarrow 4a_4 + 3a_3 = 2a_4, \quad a_4 = 0$$

$$x^4 \Rightarrow 5a_5 + 4a_4 = 2a_5, \quad a_5 = 0$$

$$y = a_0(1 + 2x + x^2)$$

$$2. y'' + y = 0$$

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots$$

$$y' = \frac{dy}{dx} = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \dots$$

$$y'' = \frac{d^2y}{dx^2} = 2a_2 + 6a_3x + 12a_4x^2 + \dots$$

$$(2a_2 + 6a_3x + 12a_4x^2 + \dots) + (a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots) = 0$$

$$(2a_2 + a_0) + (6a_3 + a_1)x + (12a_4 + a_2)x^2 + \dots = 0$$

$$x^0 = 2a_2 + a_0 = 0, \quad a_2 = -\frac{1}{2}a_0$$

$$x^1 = 6a_3 + a_1 = 0, \quad a_3 = -\frac{1}{6}a_1$$

$$x^2 = 12a_4 + a_2 = 0, \quad a_2 = -12a_4, \quad a_4 = \frac{-1}{12}a_2 = \frac{1}{24}a_0$$

$$y = a_0 + a_1x - \frac{1}{2!}a_0x^2 - \frac{1}{3!}a_1x^3 + \dots$$

$$= a_0 \cos x + a_1 \sin x$$