Quiz 6

1.$(1-x^2)y''-2xy'+n(n+1)y=0$

$P_0(x)$, $P_1(x)$, $P_2(x)$, $P_3(x)\rightarrow$

Legendre differential Eq's solution

$P_0(x)=1$ , $P_1(x)=x$ , $P_2(x)=1/2(3x^2-1)$ , $P_3(x)= 1/2(5x^3-3x)$

$P_4(x)= 1/8(35x^4-30x^2+3)$ , $P_5(x)= 1/8(63x^5-70x^3+15x)$

$$1.y = P_0(x) = 1 \Rightarrow y' = 0, y'' = 0 \Rightarrow (1-x^2)y''-2xy' = 0$$

$$n = 0 \qquad \Rightarrow (1-x^2)0 - 2x0 = 0$$

$$2.y = P_1(x) = x \Rightarrow y' = 1, y'' = 0 \Rightarrow (1-x^2)y''-2xy'+2y = 0$$

$$n = 1 \qquad \Rightarrow (1-x^2)0 - 2x1 + 2x = 0$$

$$3.y = P_2(x) = \frac{1}{2}(3x^2-1) \Rightarrow y' = 3x, y'' = 3 \Rightarrow (1-x^2)y''-2xy'+2\cdot 3y = 0$$

$$n = 0 \qquad \Rightarrow (1-x^2)3 - 2x\cdot 3x + 2\cdot 3\cdot\frac{1}{2}(3x^2-1) = 0$$

$$4.y = P_3(x) = \frac{1}{2}(5x^3-3x) \Rightarrow y' = \frac{15}{2}x^2 - \frac{3}{2}, y'' = 15x \Rightarrow (1-x^2)y''-2xy'+3\cdot 4y = 0$$

$$n = 0 \qquad \Rightarrow (1-x^2)15x - 2x\cdot(\frac{15x^2}{2} - \frac{3}{2}) + 3\cdot 4\cdot\frac{1}{2}(5x^3-3x) = 0$$

$$5.y = P_4(x)(n=4) = \frac{1}{8}(35x^4-30x^2+3) \Rightarrow y' = \frac{35}{2}x^3 - \frac{15}{2}x, y'' = \frac{105}{2}x^2 - \frac{15}{2}$$

$$\Rightarrow (1-x^2)y''-2xy'+20y$$

$$\Rightarrow (1-x^2)\frac{5}{2}(21x^2-3) - 2x\cdot\frac{1}{2}(35x^3-15x) + 20\cdot\frac{1}{8}(35x^4-30x^2+3) \neq 0$$

$$5.y = P_5(x)(n=5) = \frac{1}{8}(63x^5-70x^3+15x) \Rightarrow y' = \frac{5}{8}\cdot 63x^4 - \frac{3}{8}\cdot 70x^2 + \frac{15}{8}, y'' = \frac{5\cdot 63}{2}x^3 - \frac{35\cdot 3}{2}x$$

$$\Rightarrow (1-x^2)y''-2xy'+30y$$

$$\Rightarrow (1-x^2)(\frac{315}{2}x^3 - \frac{105}{2}x) - 2x(\frac{315}{8}x^{\backslash 4} - \frac{105}{4}x^2 + \frac{15}{8}) + (30\cdot\frac{1}{8}(63x^5-70x^3+15x) \neq 0$$