Quiz9

1.$6x^2+16xy-6y^2=20$　參考 P345

$$A=\begin{bmatrix}6 & 8\\ 8 & -6\end{bmatrix},\ x=\begin{bmatrix}x\\ y\end{bmatrix}$$

$$\begin{vmatrix}6-\lambda & 8\\ 8 & -6-\lambda\end{vmatrix}=0$$

$$(6-\lambda)(-6-\lambda)-64=0\Rightarrow\lambda=\pm10$$

$$Q=\lambda_1x^2+\lambda_2y^2=10x^2-10y^2=20\Rightarrow\frac{x^2}{\sqrt{2}^2}-\frac{y^2}{\sqrt{2}^2}=1\rightarrow\text{雙曲線}$$

2.$y_1'=y_1+y_2$, $y_2'=4y_1+y_2$　參考 P147

$y_1(t)$　$y_1(0)=1$

$y_2(t)$　$y_2(0)=6$

$$\begin{bmatrix}1 & 1\\ 4 & 1\end{bmatrix}\Rightarrow\begin{vmatrix}1-\lambda & 1\\ 4 & 1-\lambda\end{vmatrix}=(1-\lambda)^2-4=0\ \ \lambda=3,-1$$

$$\lambda_1=3$$

$$\begin{bmatrix}-2 & 1\\ 4 & -2\end{bmatrix}\Rightarrow-2x_1+x_2=0,4x_1-2x_2=0\ \begin{bmatrix}x_1\\ x_2\end{bmatrix}\Rightarrow\begin{bmatrix}1\\ 2\end{bmatrix}$$

$$\lambda_1=-1$$

$$\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix}\Rightarrow2x_1+x_2=0,4x_1+2x_2=0\ \begin{bmatrix}x_1\\ x_2\end{bmatrix}\Rightarrow\begin{bmatrix}-1\\ 2\end{bmatrix}$$

$$\text{設}y=xe^{\lambda t},y'=\lambda xe^{\lambda t}$$

$$y=C_1\begin{bmatrix}1\\ 2\end{bmatrix}e^{3t}+C_2\begin{bmatrix}-1\\ 2\end{bmatrix}e^{-t}$$

$$y_1=C_1e^{3t}-C_2e^{-t},y_1(0)=1\Rightarrow C_1-C_2=1$$

$$y_2=2C_1e^{3t}+2C_2e^{-t},y_2(0)=6\Rightarrow2C_1+2C_2=6$$

$$C_1=2,C_2=1$$

$$y=2\begin{bmatrix}1\\ 2\end{bmatrix}e^{3t}+\begin{bmatrix}-1\\ 2\end{bmatrix}e^{-t}$$