量化 Quiz 2 、2015.3.25

1、$E = nh\nu$ $(n = 0, 1, 2, \cdots\cdots)$

証：(1) $\bar{E} = \dfrac{h\nu}{e^{h\nu/kT} - 1}$ (40%)

(2) $\bar{E} \longrightarrow kT$ (high T) (10%)

$\longrightarrow 0$ (low T) (10%)

2、$\hat{H}\psi_n = E_n\psi_n$

証 $\Psi^{(x,t)} = C_1\psi_1^{(x)} e^{-iE_1 t/\hbar} + C_2\psi_2^{(x)} e^{-iE_2 t/\hbar}$ (40%)

是 $i\hbar \dfrac{\partial \Psi(x,t)}{\partial t} = \hat{H}\Psi(x,t)$ 之解．

ANS：

1、(1) (average E) = (total E)/(total number)，$E_n = nh\nu$ $(n = 0, 1, 2 \cdots)$

$$\text{total number} = N_0 + N_1 + N_2 + \cdots\cdots$$
$$= N_0(1 + e^{-\Delta E/kT} + e^{-2\Delta E/kT} \cdots\cdots)$$
$$= N_0\left(\frac{1}{1 - e^{-\Delta E/kT}}\right)$$

$$\text{total } E = \overset{0}{E_0} N_0 + E_1N_1 + E_2N_2 + \cdots\cdots$$
$$= h\nu e^{-h\nu/kT} + 2h\nu \cdot e^{-2h\nu/kT} \cdots\cdots$$

$$x = e^{-h\nu/kT} \Rightarrow h\nu x + 2h\nu x^2 + 3h\nu x^3 + \cdots\cdots$$
$$x \cdot E \Rightarrow h\nu x^2 + 2h\nu x^3 + \cdots\cdots$$
$$(1-x)E = h\nu x + h\nu x^2 + h\nu x^3 + \cdots\cdots$$
$$= h\nu\left(\frac{x}{1-x}\right)$$
$$\Rightarrow E = \frac{h\nu x}{(1-x)^2} = \frac{h\nu e^{-h\nu/kT}}{(1 - e^{-h\nu/kT})^2}$$

$$\bar{E} = \frac{E}{N} = \frac{\dfrac{h\nu e^{-h\nu/kT}}{(1-e^{-h\nu/kT})^2}}{\dfrac{1}{(1-e^{-h\nu/kT})}} = \frac{h\nu e^{-h\nu/kT}}{1 - e^{-h\nu/kT}} = \frac{h\nu}{e^{h\nu/kT} - 1} \quad //$$

(2) 泰勒展開：$e^x = 1 + x + \cancel{\frac{1}{2!}x^2 + \cdots}$

if $x \ll 1$，高次項可忽略

high T：$\bar{E} = \frac{h\nu}{e^{h\nu/kT}-1} = \frac{h\nu}{\cancel{1}+\frac{h\nu}{kT}-\cancel{1}} = kT$ //

low T：$\frac{h\nu}{e^{h\nu/kT}-1} \doteqdot h\nu e^{-h\nu/kT} \xrightarrow{(T\to 0)} 0$ //

2、

左式 $= i\hbar\frac{\partial}{\partial t}\left[C_1\psi_1^{(x)}e^{-iE_1t/\hbar} + C_2\psi_2^{(x)}e^{-iE_2t/\hbar}\right] = i\hbar\left[C_1\psi_1^{(x)}\frac{\partial}{\partial t}e^{-iE_1t/\hbar} + C_2\psi_2^{(x)}\frac{\partial}{\partial t}e^{-iE_2t/\hbar}\right]$

$= i\hbar\cdot\frac{-i}{\hbar}\left[E_1C_1\psi_1^{(x)}e^{-iE_1t/\hbar} + E_2C_2\psi_2^{(x)}e^{-iE_2t/\hbar}\right] = E_1C_1\psi_1^{(x)}e^{-iE_1t/\hbar} + E_2C_2\psi_2^{(x)}e^{-iE_2t/\hbar}$ //

右式 $= H\Psi(x,t) = C_1\underline{\hat{H}\psi_1^{(x)}}e^{-iE_1t/\hbar} + C_2\underline{\hat{H}\psi_2^{(x)}}e^{-iE_2t/\hbar}$ $(\hat{H}\psi_n = E_n\psi_n)$

$= C_1E_1\psi_1^{(x)}e^{-iE_1t/\hbar} + C_2E_2\psi_2^{(x)}e^{-iE_2t/\hbar}$ //

∵ 左式 = 右式

∴ $\Psi^{(x,t)} = C_1\psi_1^{(x)}e^{-iE_1t/\hbar} + C_2\psi_2^{(x)}e^{-iE_2t/\hbar}$

為此 $i\hbar\frac{\partial\Psi(x,t)}{\partial t} = \hat{H}\Psi(x,t)$ equation 之解